

Quantum Communication Complexity using the Quantum Zeno Effect

Armin Tavakoli,¹ Hammad Anwer,¹ Alley Hameedi,¹ and Mohamed Bourennane¹

¹*Department of Physics, Stockholm University, S-10691 Stockholm, Sweden*

The quantum Zeno effect (QZE) is the phenomenon where the unitary evolution of a quantum state is suppressed e.g. due to frequent measurements. Here, we investigate the use of the QZE in a class of communication complexity problems (CCPs). Quantum entanglement is known to solve certain CCPs beyond classical constraints. However, recent developments have yielded CCPs where super-classical results can be obtained using only communication of a single d -level quantum state (qudit) as a resource. In the class of CCPs considered here, we show quantum reduction of complexity in three ways: using i) entanglement and the QZE, ii) single qudit and the QZE, iii) single qudit. The final protocol is motivated by experimental feasibility, and we have performed a proof of concept experimental demonstration.

I. INTRODUCTION

The quantum Zeno effect (QZE) is the phenomenon where the unitary evolution of a quantum state can be suppressed e.g. by interactions with the environment or frequently performing measurements on the state [1, 2]. The QZE is of interest in various fields and topics in physics, including decaying systems, trapped cold atoms, quantum computation, (nonlinear) optics and quantum foundations [3–10]. Additionally, the QZE has been implemented in a quantum protocol using entanglement to improve a communication complexity problem (CCP) beyond classical limitation [11].

A CCP is typically described by two parties Alice and Bob who aim to compute the value of a function $f(x, y)$, depending on bit-strings x, y , one given to Alice and one to Bob, despite restricted communication. The task is to maximize the probability that Bob computes f , given that Alice may communicate no more than k bits to Bob. Naturally, this allows for various generalizations e.g. CCPs involving many parties or high-level inputs.

It is well known that quantum entanglement can give rise to correlations that do not admit a local model [12] and that such correlations can be implemented with local information processing to improve CCPs beyond classical protocols [13–16]. Close links have been established between Bell inequality violation and reduction of communication complexity [14, 17, 18], providing a fundamental understanding for why quantum entanglement achieves complexity reduction.

Interestingly, it has been shown that quantum reduction of communication complexity can be achieved without shared entangled states. In [19], quantum strategies for two particular CCPs, relying only on sequential communication of a single quantum two-level system (qubit), were shown to lower complexity beyond classical limitations. Indeed, the superiority of these quantum strategies can no longer be explained by violations of Bell inequalities. Also in other communications tasks, where the quantum advantages previously stemmed from Bell inequality violations, has it been shown that the single qubit can be used to outperform classical protocols [20]. The single qubit protocols are not only conceptually in-

teresting but also tend to be more scalable than the protocols based on entanglement, making them feasible for experimental implementations.

In this paper, we introduce a family of CCPs, generalizing the particular CCP in [11], and investigate classical and quantum solutions. In particular, we aim to answer two questions (1) how does entanglement and a single quantum d -level system (qudit) perform as resources for reduction of communication complexity?, and (2) do protocols that use the QZE perform better than protocols that do not? In our investigation, we will provide three quantum solutions to our CCPs, (i) using entanglement and the QZE, (ii) using a single qudit and the QZE, and finally (iii) using single qudit without the QZE. The final protocol is argued to be experimentally feasible and scalable, and we will provide an experimental demonstration of quantum reduction of communication complexity.

II. SOLVING THE COMMUNICATION COMPLEXITY PROBLEMS

The class of CCPs we consider is described as follows: imagine that a distributor supplies M parties R_1, \dots, R_M with inputs in such a way that R_1 and R_M are given x_1 and y_M respectively, whereas parties R_2, \dots, R_{M-1} are given two inputs each, x_i, y_i . All inputs are elements of the set $\{0, \dots, dN - 1\}$ where $d > 1$ and N is a large integer, which are publicly announced by the distributor. In addition, for $l = 0, \dots, d - 1$ define sets $S_l = \{Nl - \mu, \dots, Nl + \mu\}$, where addition is taken modulo dN , with some publicly announced positive integer μ such that $N \gg \mu$. The distributor promises the parties that each pair of inputs x_i, y_{i+1} obeys $x_i - y_{i+1} \in S_l$ for some l . The CCP facing the M parties is for R_i to communicate at most one d -level of information to R_{i+1} for $i = 1, \dots, M - 1$, in such a way that R_M with high probability can announce a value $l' \in \{0, \dots, d - 1\}$ such that $\sum_{i=1}^{M-1} x_i - y_{i+1} \in S_{l'}$. Thus, each CCP is characterized by the numbers (N, M, d, μ) .

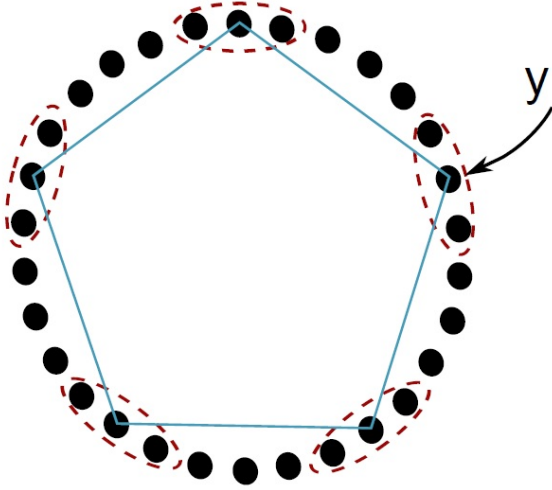


FIG. 1. The dN numbers can be imagined as dots on a circle. For given y , the allowed values of x are given by the μ numbers on either side of the dots that constitutes vertices of the regular convex polygon with one vertex at y . Here we have assumed $N = 6$, $\mu = 1$ and $d = 5$.

A. Classical solution

Let us begin with considering classical strategies for our family of CCPs. This problem was considered for $d = 2$ and $\mu = 1$ in [11] and we now solve the more general classical problem. Firstly, let us consider only two parties involved. Since the distribution of the inputs in the CCP are given on advance, we can without loss of generality restrict to deterministic classical strategies, i.e. strategies that are not randomized according to some pre-established rule. R_1 must then use a function $f : \{0, \dots, dN - 1\} \rightarrow \{0, \dots, d - 1\}$ to determine the value of the classical d -level that is sent to R_2 . The dN possible values of the input x can be represented as dots forming a circle. Due to the distributor's promise, for a given input y of R_2 , there are only $d(2\mu + 1)$ allowed possible values of x , all equally probable. For given y , the possible values of x can be represented by the μ dots on either side of the particular elements constituting vertices of the regular convex polygon with d sides that has one vertex on the dot representing y , see figure (1). R_1 divides the circle into d equal parts, each containing N subsequent dots. This division can be made arbitrarily since the distribution of the inputs is uniform. For simplicity, make divisions $G_k = \{Nk, \dots, N(k+1) - 1\}$ for $k = 0, \dots, d - 1$. R_1 uses $f(x) = k$ if and only if $x \in G_k$. It is evident that whenever all the elements $y - \mu, \dots, y + \mu \in G_k$ for some k , $f(x)$ will always give R_2 the information necessary to with certainty find l' such that $x - y \in S_{l'}$. However, whenever not all of the elements $y - \mu, \dots, y + \mu$ are members of G_k for some k , then there is a probability that R_2 does not manage to find l' . The error rate depends on how many of the $2\mu + 1$ numbers that are inside some

G_k . Evidently, if we have the first $q \in \{\mu + 1, \dots, 2\mu + 1\}$ numbers in G_k and the final $2\mu + 1 - q$ numbers in G_{k+1} , R_2 will guess that $x \in G_k$ and have a probability of $\frac{q}{2\mu + 1}$ to succeed. From the dN possible values of y , only $2d\mu$ choices introduce an error. Therefore, the average probability of error is found from

$$P_{error}^C = \frac{1}{dN} \left(2d \sum_{i=1}^{\mu} \left(1 - \frac{\mu + i}{2\mu + 1} \right) \right) = \frac{\mu(\mu + 1)}{N(2\mu + 1)} \quad (1)$$

This constitutes the error probability for two parties executing the protocol. However, in the classical strategy, this procedure is simply repeated $M - 1$ times, once between each pair R_i and R_{i+1} of the M parties, with each party R_i adding to his input the received value of its previous execution of the protocol with R_{i-1} . The protocol succeeds whenever there is either no error in any of the executions, or if the errors cancel each other out. For large $N \gg M$, we can neglect the possibility of canceling errors and thus the error probability for M parties could be estimated by

$$P_{error}^C(M) \approx \frac{\mu(\mu + 1)(M - 1)}{N(2\mu + 1)} \quad (2)$$

We can conclude that the probability of error is proportional to $1/N$. However, if we have $M > N$, we cannot neglect the possibility of errors canceling so (2) breaks down. In analogy with the argument in [11], the success probability the protocol drops to the vicinity of $1/d$ which is the trivial strategy where R_M takes a random guess on the answer to the CCP. To our knowledge, a proof of the optimal classical strategy is not known.

B. Quantum solution with entanglement

Let us investigate quantum strategies, and firstly let consider the protocol based on entanglement which exploits the QZE and let us called it P_E . This protocol is the generalization of the know result of [11]. Let R_1 and R_2 share the entangled state $|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |kk\rangle$. R_1 will use his input x_1 to locally perform a unitary transformation $U(x_1)$ on his part of the shared state.

$$U(x_1) = \sum_{k=0}^{d-1} \omega^{\frac{x_1}{N}k} |k\rangle\langle k| \quad (3)$$

where $\omega = e^{\frac{2\pi i}{d}}$. Similarly, R_2 locally performs $U(-y_2)$. Then, R_1 and R_2 perform local measurements of their part of the state in the Fourier basis given by $|e_l\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \omega^{kl} |k\rangle$. The resulting probability distribution over the respective outcomes $l_1^{(R)}$ and $l_2^{(L)}$ of R_1 and R_2

is

$$P(l_1^{(R)}, l_2^{(L)}) = \frac{1}{d^3} \left[\left(\sum_{k=0}^{d-1} \cos \left(\frac{2\pi k}{d} \left(\frac{x_1 - y_2}{N} - l_1^{(R)} - l_2^{(L)} \right) \right) \right)^2 + \left(\sum_{k=0}^{d-1} \sin \left(\frac{2\pi k}{d} \left(\frac{x_1 - y_2}{N} - l_1^{(R)} - l_2^{(L)} \right) \right) \right)^2 \right] \quad (4)$$

However, due to the promise from the distributor we can write $x_1 - y_2 = aN + b$ with $a \in \{0, \dots, d-1\}$ and $b \in \{-\mu, \dots, \mu\}$. Since $\mu \ll N$, outcomes of the form $l_1^{(R)} + l_2^{(L)} = a \bmod d$ occur with high probability, and we can choose $l_1^{(R)}, l_2^{(L)}$ in d different ways such that this relation is satisfied. To bound $P(l_1^{(R)} + l_2^{(L)} = a)$ from below, we neglect the contribution from the sine terms, put $b = \pm\mu$, decouple the cosine expression from the sum by putting $k = d-1$ and use the small angle approximation $\cos z \approx 1 - z^2/2$, thus finding

$$P(l_1^{(R)} + l_2^{(L)} = a) \geq \cos^2 \left(\frac{2\pi\mu(d-1)}{dN} \right) \approx 1 - \frac{4\pi^2\mu^2(d-1)^2}{d^2N^2} \quad (5)$$

Hence, let R_1 communicate his outcome $l_1^{(R)}$ to R_2 , who computes $l_1^{(R)} + l_2^{(L)} \bmod d$ and then with a high probability knows that $x_1 - y_2 \in S_{l_1^{(R)} + l_2^{(L)}}$. This accounts for the first step in the protocol. However, it is easily realized that each pair of subsequent parties R_i and R_{i+1} can use the above protocol with their inputs x_i, y_{i+1} and R_i adding his outcome $l_i^{(R)}$ to the message received from R_{i-1} and communicating this to R_{i+1} . Thus, after the required $M-1$ executions of the protocol, R_M will be able to announce the correct value l' such that $\sum_{i=1}^{M-1} x_i - y_{i+1} \in S_{l'}$ if and only if there was either no error in any of the $M-1$ executions, or if the errors cancel each other out. However, for errors to cancel there must be at least two failed steps. Similar to the above classical case, we consider large $N \gg M$ and thus neglect cases with more than one error occurring allowing us to approximate the success probability as $P(l_1^{(R)} + l_2^{(L)} = a)^{M-1}$. To good accuracy, we can bound the quantum success probability from below by considering only the two first terms i.e.

$$P_{ent}^Q \geq 1 - \frac{4\pi^2\mu^2(d-1)^2(M-1)}{d^2N^2} \quad (6)$$

This shows that the error probability in the CCP characterized by the tuple (N, M, d, μ) is proportional to $1/N^2$, thus lowering communication complexity beyond what was achieved with the classical protocol.

C. Quantum solution with single qudit

Let us now consider quantum solutions to the CCP using only a single qudit. Our protocol P_1 will rely on sequential communication of a single qudit with only the final recipient performing a measurement. Thus, P_1 is not a manifestation of the QZE since we are not suppressing the unitary evolution with repeated measurements. Nevertheless, we will see that complexity can be reduced beyond classical limitations.

Let R_1 prepare the uniform superposition state $|\psi_0\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle$. Now, R_1 acts with the local unitary $U(x_1)$ (used in protocol P_E) on $|\psi_0\rangle$ and then communicates the qudit to R_2 . R_2 transforms the state by applying $U(x_2 - y_2)$ and communicates the qudit to R_3 . Parties R_3, \dots, R_{M-1} act in analogy with R_2 . When the quantum system is given to R_M , he performs $U(-y_M)$. The final state of the system is $|\psi_M\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \omega^{\frac{k}{N} \sum_{i=1}^{M-1} (x_i - y_{i+1})} |k\rangle$. Finally, R_M performs a measurement on the qudit in the Fourier basis. The outcome of R_M 's measurement is labeled l and is subject to a probability distribution

$$P(l) = \frac{1}{d^2} \left[\left(\sum_{k=0}^{d-1} \cos \left(\frac{2\pi k}{d} \left(\frac{1}{N} \sum_{i=1}^{M-1} (x_i - y_{i+1}) - l \right) \right) \right)^2 + \left(\sum_{k=0}^{d-1} \sin \left(\frac{2\pi k}{d} \left(\frac{1}{N} \sum_{i=1}^{M-1} (x_i - y_{i+1}) - l \right) \right) \right)^2 \right] \quad (7)$$

Due to the distributor's promise, we write $x_i - y_{i+1} = a_i N + b_i$ with $a_i \in \{0, \dots, d-1\}$ and $b_i \in \{-\mu, \dots, \mu\}$. Hence we can write $\frac{1}{N} \sum_{i=1}^{M-1} (x_i - y_{i+1}) = \sum_{i=1}^{M-1} (a_i + \frac{1}{N} b_i) = A + \frac{B}{N}$ where we have let $\sum a_i \equiv A$ and $\sum b_i \equiv B$. Thus, we have $A \in \{0, \dots, (d-1)(M-1)\}$ and $B \in \{-\mu(M-1), \dots, \mu(M-1)\}$. However, A can be reduced modulo d without loss of generality. The CCP is successfully solved if R_M finds $l = A \bmod d$. Applying approximations similar to what was used to obtain (5), we find that $P_{qudit}^Q \equiv P(l = A \bmod d)$

$$P_{qudit}^Q \geq 1 - \frac{4\pi^2(d-1)^2\mu^2(M-1)^2}{d^2N^2} \quad (8)$$

Indeed, the protocol P_1 can reduce communication complexity beyond the classical protocol. The above lower bound on success probability differs from (6) obtained using entanglement by being quadratic in $M-1$. However, in cases where N is sufficiently larger than M , the difference between (8) and (6) becomes negligible. However, the drop in success probability should not be interpreted as originating from the use of a single qudit instead of entanglement as a resource, but rather as consequence of not using the QZE to suppress the unitary evolution of the state. This is a manifestation of the QZE contributing to reduction of communication complexity, in addition to the quantum resource. To support this claim,

we will now construct a single qudit protocol, P_2 , for the family of CCPs that reproduces the success probability in (6) by invoking the QZE.

In P_2 , R_1 prepares the same state as in P_1 , that is: $|\psi_0\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle$, performs the unitary action $U(x_1)$, and sends the qudit to R_2 who performs $U(-y_2)$. R_2 then measures the qudit in the Fourier basis using a quantum non-demolition (QND) measurement. This renders the system in one of the d states forming the Fourier basis. Then, R_2 applies $U(x_2)$ to the system and communicates the state to R_3 . Parties R_3, \dots, R_{M-1} act in analogy with R_2 i.e. each will perform a unitary rotation, a QND measurement in the Fourier basis, and finally another unitary rotation after which they communicate the system to the subsequent party. When R_M obtains the qudit, he performs $U(-y_M)$ and then measures (need not be a QND measurement) the system in the Fourier basis, and uses the outcome to with high probability solve the CCP. The use of the QZE plays an important role in the efficiency of protocol P_2 . Since every pair of neighboring parties perform some unitaries on the system and then do a QND measurement, we can imagine P_2 as each pair of neighbors running the protocol P_1 throughout the line of all M parties. That is, the probability of the outcome of the first QND measurement, performed by the second party in the protocol, to be associated to a successful outcome amounts to putting $M = 2$ in (8). Since with $M = 2$, (8) becomes equivalent to (6), the success probability of P_2 when run for M parties becomes the equivalent to what was obtained using entanglement by invoking the same approximations.

D. Comparing the protocols

Let us now compare the strengths and weaknesses of the three quantum protocols. Indeed, since P_1 does not exploit the QZE, it performs worse than the other two protocols. However, both single qudit protocols but in particular P_1 , are subject to various advantages over the protocol P_E using entanglement. Firstly, P_E requires the preparation and distribution of $M - 1$ two-qudit entangled states whereas in P_1 and P_2 only the preparation of the uniform superposition is required. Secondly, P_E requires $2(M - 1)$ measurements, P_2 requires $M - 1$ measurements, whereas P_1 requires only one single measurement independent of M . In the highly realistic case of parties having non-ideal detectors with efficiency $\eta \in [0, 1]$, it is sufficient that one single measurement fails in order for the success probability to drop to the vicinity of $1/d$, which can be reproduced by guessing. The probability of all measurements succeeding in P_E is $\eta^{2(M-1)}$ and in P_2 it is η^{M-1} , both rapidly decreasing as M increases. However, using P_1 , the probability of successful detection is constantly η . These experimental advantages make P_1 an experimentally feasible and scalable protocol. In table (I) we list the properties of our protocols P_E, P_1 and P_2 .

-	P_E	P_1	P_2
Quantum resource	entanglement	single qudit	single qudit
Use of QZE	Yes	No	Yes
Protocol efficiency	$\eta^{2(M-1)}$	η	η^{M-1}
Probability of failure	$\propto M/N^2$	$\propto M^2/N^2$	$\propto M/N^2$

TABLE I. Review of the properties of the three quantum protocols P_E, P_1 and P_2 solving the family of CCPs.

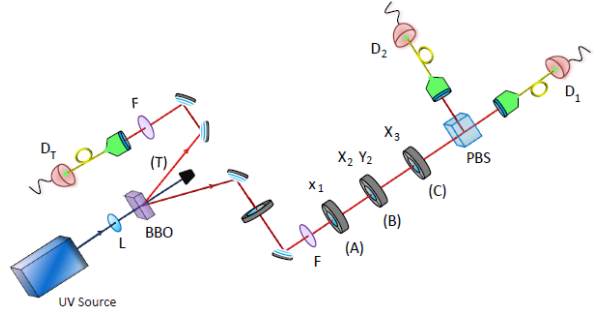


FIG. 2. Experimental setup for the protocol P_1 . Heralded single photon source consists of SPDC process, a focused (with lens L) UV light source pumping a BBO nonlinear crystal, the converted photons are emitted in two spatial modes, pass a filter (F) and coupled to single mode fiber. The idler photon is used as trigger and detected by a single photon detector D_T . Alice, Bob, Charlie perform their action x_1, y_2 and x_2, y_3 by rotating half wave plates (HWP). The polarization measurement consist of a polarization beam splitter (PBS) and two single photon detectors D_1 and D_2 . These detectors are Si avalanche photodiodes (APD)

III. EXPERIMENTAL REALIZATION

We will now experimentally implement our protocols P_1 and P_2 . For the experimental proof of principle we have implemented the CCP protocols with $(N, M, d, \mu, \nu) = (60, 3, 2, 1)$ i.e. for three parties (Alice, Bob, and Charlie) using single qubit communication. In our experiment, the physical systems are defined by single photons in a polarization setup. The basis vectors $|0\rangle$, and $|1\rangle$ correspond to finding the photon in horizontal or vertical polarization respectively. Single photons are generated from a heralded single photon source through a spontaneous parametric down-conversion (SPDC) process. The idler photon is used as trigger and detected by a single photon detector D_T . To exactly define the spatial and spectral properties of the signal photon, the emitted photon modes are coupled into a single mode fiber (SMF) and passed through a narrowband interference filter (F). In the experimental realization, we work in the xz -plane of the Bloch sphere instead of the xy -plane used in the presented theory. Thus, we prepare the initial photon in $|H\rangle$, the signal photon is passing through a polarizer oriented to horizontal polarization direction.

To execute their actions x_1, y_2 and x_2, y_3 , Al-

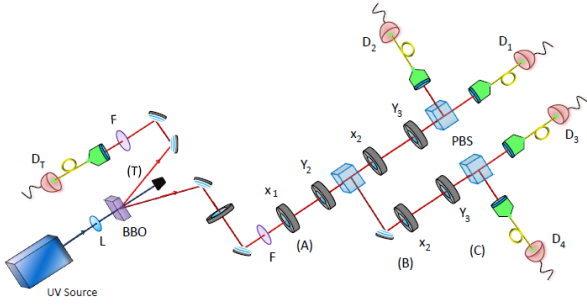


FIG. 3. Experimental setup for the protocol P_2 . The same heralded single photon source as for P_1 . Alice, Bob, Charlie perform their action x_1 , y_2 and x_2 , and y_3 by rotating half wave plates (HWP). The Bob's quantum non-demolition (QND) measurement consists of a polarization beam splitter (PBS) where the outcome of this polarization measurement is encoded in the path of the photon. The polarization measurement consist of two polarization beam splitters (PBS) and four single photon detectors D_1 , D_2 , D_3 , and D_4

ice, Bob, Charlie perform sequentially unitary transformations on the incoming qubit respectively: a rotation about the y -axis of the Bloch sphere with angle $\theta = \frac{\pi z_1}{N}$ where $z_1 \in \{x_1, x_2+y_2, y_3\}$ for the protocol P_1 (see Fig. 2) and with angle $\theta = \frac{\pi z_2}{N}$ where $z_2 \in \{x_1, y_2, x_2, y_3\}$ for the protocol P_2 (see Fig. 2). These transformations are achieved by rotating the polarization of the single photon with help of half wave plates (HWP). Bob's quantum non-demolition (QND) measurement, necessary for protocol P_2 , is performed with a polarization beam splitter (PBS) where the outcomes of this measurement (that only addresses the polarization degrees of freedom) is encoded in the path of the photon. Finally, the measurement consists of a PBS and two single photon detectors D_1 and D_2 and two PBS and four single photon detectors D_1 , D_2 , D_3 and D_4 for the protocols P_1 and P_2 respectively. The D_i detectors ($i = 1, 2, 3, 4, T$) are Si avalanche photodiodes (APD). All coincidence counts between the signal and idler photons are registered using an three-channel coincidence logic with a time window of 1.7 ns. The number of detected photons was approximately 9.1×10^4 per second and the total time used for each experimental settings was 60 s. The experimental results are presented in Table II. The errors come from Poissonian counting statistics and systematic errors. Due to the high photon counts, the Poissonian errors are negligible. The main sources of systematic errors are the slight intrinsic imperfections of the PBSs and HWPs.

The results are in very good agreement with the predictions of quantum mechanics. For an ideal experiment with protocol P_1 (P_2) the quantum success probability is $P_Q \geq 0.9890$ ($P_Q \geq 0.9945$) whereas the classical success probability is $P_C = 0.9778$. Our data gives the average quantum success probability $P_Q^{exp,1} \approx 0.9914 \pm 0.0003$ for protocol P_1 and $P_Q^{exp,2} \approx 0.9921 \pm 0.0003$ for protocol P_2 . Both averages are clearly above the classical bound and

cNo	x_1	y_2	x_2	y_3	$P_{success}^{exp,1}$	$P_{success}^{exp,2}$
1	70	71	55	56	0.9944 ± 0.0004	0.9927 ± 0.0021
2	58	59	38	37	0.9800 ± 0.0035	0.9951 ± 0.0021
3	67	7	88	29	0.9864 ± 0.0019	0.9894 ± 0.0015
4	40	101	15	76	0.9904 ± 0.0004	0.9948 ± 0.0021
5	70	10	40	101	0.9967 ± 0.0019	0.9918 ± 0.0015
6	36	36	117	56	0.9812 ± 0.0019	0.9896 ± 0.0015
7	44	103	117	57	0.9965 ± 0.0018	0.9900 ± 0.0015
8	80	19	108	108	0.9983 ± 0.0019	0.9931 ± 0.0015
9	36	36	38	37	0.9797 ± 0.0019	0.9951 ± 0.0015
10	117	57	72	13	0.9814 ± 0.0018	0.9915 ± 0.0015
11	0	0	63	4	0.9884 ± 0.0018	0.9927 ± 0.0015
12	17	18	67	67	0.9852 ± 0.0018	0.9980 ± 0.0015
13	99	40	19	80	0.9963 ± 0.0004	0.9934 ± 0.0021
14	79	18	80	79	0.9900 ± 0.0004	0.9869 ± 0.0021
15	48	108	69	10	0.9978 ± 0.0019	0.9939 ± 0.0016
16	104	105	8	7	0.9836 ± 0.0036	0.9897 ± 0.0021
17	25	25	20	80	0.9902 ± 0.0004	0.9956 ± 0.0003
18	33	94	59	118	0.9878 ± 0.0036	0.9960 ± 0.0022
19	87	28	40	100	0.9849 ± 0.0018	0.9957 ± 0.0016
20	63	3	98	38	0.9926 ± 0.0004	0.9914 ± 0.0003
21	119	58	115	54	0.9973 ± 0.0004	0.9900 ± 0.0021
22	110	50	101	41	0.9971 ± 0.0004	0.9915 ± 0.0004
23	64	3	58	59	0.9991 ± 0.0038	0.9932 ± 0.0020
24	82	81	94	33	0.9909 ± 0.0004	0.9840 ± 0.0022
25	60	0	0	119	0.9843 ± 0.0018	0.9920 ± 0.0015
26	60	60	15	16	0.9989 ± 0.0019	0.9893 ± 0.0015
27	108	47	119	0	0.9948 ± 0.0038	0.9937 ± 0.0022
28	94	33	14	13	0.9984 ± 0.0004	0.9939 ± 0.0021
29	114	55	7	8	0.9974 ± 0.0004	0.9915 ± 0.0021
30	109	49	9	8	0.9879 ± 0.0020	0.9912 ± 0.0016
31	103	103	90	29	0.9962 ± 0.0019	0.9915 ± 0.0015
32	74	73	24	85	0.9980 ± 0.0037	0.9902 ± 0.0021
33	2	63	28	28	0.9877 ± 0.0018	0.9852 ± 0.0016
34	109	49	69	8	0.9852 ± 0.0019	0.9925 ± 0.0015
35	7	7	44	44	0.9874 ± 0.0004	0.9923 ± 0.0003
36	110	50	90	30	0.9960 ± 0.0004	0.9901 ± 0.0004
37	56	56	5	64	0.9849 ± 0.0019	0.9914 ± 0.0015
38	9	9	11	71	0.9871 ± 0.0004	0.9965 ± 0.0003
39	48	49	6	67	0.9946 ± 0.0004	0.9953 ± 0.0020
40	2	2	98	97	0.9891 ± 0.0019	0.9907 ± 0.0015

TABLE II. Experimental results for the success probability with inputs x_1 , y_2 and x_2 , and y_3 for Alice, Bob, and Charlie respectively. By $P_{success}^{exp,1}$ ($P_{success}^{exp,2}$) we denote the measured results for protocol P_1 (P_2).

our experiment demonstrates the advantage of protocol P_2 over P_1 .

IV. CONCLUSIONS

In this paper, we have investigated classical and quantum solutions for a family of CCPs. We provided a classical solution and proposed three different quantum protocols improving the CCPs beyond the classical performance. Two of the quantum protocols use QZE, one relying on entanglement as a resource while the other relying on single qudit communication, and we showed that the performance of both protocols is equal. We also proposed a protocol with single qudit communication with-

out the QZE, and the performance was shown to be lower than the other two quantum protocols. We gave a proof of concept experimental demonstration of reduction of communication complexity beyond the classical bound for both the single qudit protocol using the QZE and the protocol that does not use the QZE. Our experimental findings demonstrated the advantages of using the protocol based on the QZE. Our results suggest that the use of the QZE together with quantum resources could enhance information processing in certain tasks in comparison to quantum protocols not using the QZE.

This project was supported by the Swedish Research Council, ADOPT, and QOLAPS project of ERC.

-
- [1] B. Misra and E. C. G. Sudarshan, *The Zeno's Paradox in Quantum Theory*, J. Math. Phys. **18**, 756 (1977).
 - [2] A. Degasperis, L. Fonda and G. C. Ghirardi, *Does the lifetime of an unstable system depend on the measuring apparatus?*, II Nuovo Cimento A **21**, 3 pp. 471-484 (1974).
 - [3] S. R. Wilkinson, et al., *Experimental evidence for non-exponential decay in quantum tunnelling*, Nature **387**, 575-577 (1997).
 - [4] M. C. Fisher, B. Gutierrez-Medina and M. G. Raizen, *Observation of the quantum Zeno and anti-Zeno effects in an unstable system*, Phys. Rev. Lett. **87**, 040402 (2001).
 - [5] E. W. Streed, et al., *Continuous and pulsed quantum Zeno effect* Phys. Rev. Lett. **97**, 260402 (2006).
 - [6] P. Facchi and S. Pascazio, *Quantum Zeno subspaces*, Phys. Rev. Lett. **89**, 080401 (2002).
 - [7] J. D. Franson, B. C. Jacobs and T. B. Pittman, *Quantum computing using single photons and the Zeno effect*, Phys. Rev. A **70**, 062302 (2004).
 - [8] K. Yamane, M. Ito and M. Kitano, *Quantum Zeno effect in optical fibers*, Optics Communications **192**, 3-6 (2001).
 - [9] J. Perina, *Quantum Zeno effect in cascaded parametric down-conversion with losses*, Optics Communications **325**, 1 (2004).
 - [10] D. Home and M. A. B. Whitaker, *The many-worlds and relative states interpretations of quantum mechanics, and the quantum Zeno paradox*, J. Phys A: Math. Gen. **20** 3339 (1987).
 - [11] L. Hardy and W. van Dam, *Quantum communication using a nonlocal Zeno effect*, Phys. Rev. A **59**, 2635 (1999).
 - [12] J. S. Bell, *On the Einstein Podolsky Rosen paradox*, Physics, **1**, 195 (1964).
 - [13] R. Cleve and H. Buhrman, *Substituting quantum entanglement for communication*, Phys. Rev. A **56**, 1201 (1997).
 - [14] H. Buhrman, R. Cleve, S. Massar and R. de Wolf, *Non-locality and communication complexity*, Rev. Mod. Phys. **82**, 665 (2010).
 - [15] H. Buhrman, R. Cleve and W. van Dam, *Quantum entanglement and communication complexity*, SIAM J. Comput **30** 1829-1841 (2001).
 - [16] H. Buhrman, W. van Dam, P. Hoyer and A. Tapp, *Multiparty quantum communication complexity*, Phys. Rev. A **60**, 2737 (1999).
 - [17] C. Brukner, M. Zukowski and A. Zeilinger, *Quantum communication complexity protocol with two entangled qutrits*, Phys. Rev. Lett. **89**, 197901 (2002).
 - [18] C. Brukner, M. Zukowski, J.-W. Pan and A. Zeilinger, *Bells inequalities and quantum communication complexity*, Phys. Rev. Lett. **92**, 127901 (2004).
 - [19] P. Trojek, C. Schmid, M. Bourennane, C. Brukner, M. Zukowski and H. Weinfurter, *Experimental quantum communication complexity*, Phys. Rev. A **72**, 050305(R) (2005).
 - [20] C. Schmid, P. Trojek, H. Weinfurter, M. Bourennane, M. Zukowski and C. Kurtsiefer, *Experimental single qubit quantum secret sharing*, Phys. Rev. Lett. **95**, 230505 (2005).